RPiS - LISTA 6

CD. 10.:

Gęstość brzegowa X_1:

$$h_{X_1}(x_1) = \int_{-\sqrt{1-x_1^2}}^{\sqrt{1-x_1^2}} \frac{1}{\pi}\, dx_2 = \frac{1}{\pi}\left(\sqrt{1-x_1^2} + \sqrt{1-x_1^2}\right) = \frac{2\sqrt{1-x_1^2}}{\pi}$$

X_2:

$$h_{X_2}(x_2) = \int_{-1}^{1} \frac{1}{\pi}\, dx_1 = \frac{1}{\pi}(1+1) = \frac{2}{\pi}$$

11.

Najpierw zależność:

$$h_{X_1}(x_1) \cdot h_{X_2}(x_2) = \frac{2\sqrt{1-x_1^2}}{\pi} \cdot \frac{2}{\pi} = \frac{4\sqrt{1-x_1^2}}{\pi^2} \neq \frac{1}{\pi} = h(x_1, x_2)$$

Teraz pokażemy, że wsp. korelacji X_1 i X_2 jest zerem. $\rho = \frac{Cov(X_1, X_2)}{\sigma_{X_1} \cdot \sigma_{X_2}}$. Wystarczy pokazać, że kowariancja jest równa 0.

$$Cov(X_1, X_2) = E\left((X_1 - EX_1)\cdot(X_2 - EX_2)\right) = E(X_1 X_2) - E(X_1)\cdot E(X_2) = m_{11} - m_{10}\cdot m_{01}$$

[przypomnienie: $m_{kl} = E(X_1^k X_2^l) = \int_{X_1}\int_{X_2} x_1^k x_2^l\, h(x_1, x_2)\, dx_2\, dx_1$]

$$m_{11} = \int_{-1}^{1}\int_{-\sqrt{1-x_1^2}}^{\sqrt{1-x_1^2}} \frac{x_1 x_2}{\pi}\, dx_2\, dx_1 = \frac{1}{\pi}\left(\int_{-1}^{1} x_1 \left[\frac{x_2^2}{2}\right]_{-\sqrt{1-x_1^2}}^{\sqrt{1-x_1^2}} dx_1\right) = \frac{1}{\pi}\left(\int_{-1}^{1} x_1 \cdot \left(\frac{1-x_1^2-1+x_1^2}{2}\right) dx_1 =\right.$$

$$\frac{1}{\pi}\int_{-1}^{1} 0\, dx_1 = 0$$

$$m_{10} = \int_{-1}^{1}\int_{-\sqrt{1-x_1^2}}^{\sqrt{1-x_1^2}} \frac{x_1}{\pi}\, dx_2\, dx_1 = \frac{2}{\pi}\int_{-1}^{1} x_1\sqrt{1-x_1^2}\, dx_1$$ = Nyaaah! Lepiej m_{01}.

$$m_{01} = \int_{-1}^{1}\int_{-\sqrt{1-x_1^2}}^{\sqrt{1-x_1^2}} \frac{x_2}{\pi}\, dx_2\, dx_1 = \frac{1}{\pi}\int_{-1}^{1}\left[\frac{x_2^2}{2}\right]_{-\sqrt{1-x_1^2}}^{\sqrt{1-x_1^2}} dx_1 = \frac{1}{\pi}\int_{-1}^{1} 0\, dx_1 = 0$$

Czyli: $Cov(X_1, X_2) = m_{11} - m_{10}\cdot m_{01} = 0 - m_{10}\cdot 0 = 0 \Rightarrow \rho(X_1, X_2) = 0$