Programming 2009 — international version

Basic exam

June 30, 2009

Duration of the exam: 180 minutes. Grades:

points	grade
0-8	2.0
9–10	3.0
11-12	3.5
13–14	4.0
15–16	4.5
17	5.0

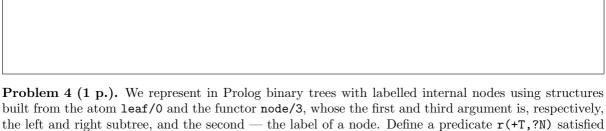
Problem 1 (1 p.). Define in Prolog a predicate p(?L) satisfied when the remainder of the division of the length of the list L by 4 is 3. You are not allowed to use arithmetic or auxiliary predicates.

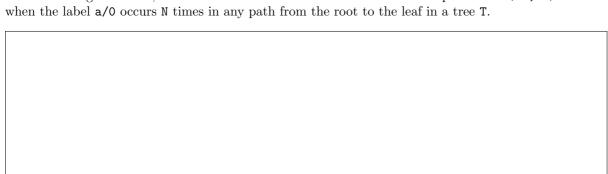
Problem 2 (1 p.). Define in Prolog a predicate revapp/3 that acts similarly to the standard predicate append/3 with the difference that the first list is reversed before it is appended to the second one. We have for example:

?- revapp([1,2,3],[4,5],
$$X$$
).
 $X = [3, 2, 1, 4, 5]$.

The predicate should work correctly at least in the mode revap(+L1,+L2,?L3). You are not allowed to use arithmetic or auxiliary predicates.

Problem	3 (1 p.). W	Ve represent	in Prolog	binary repr	esentation	ns of nonn	egative	integers	using	lists
of digits $\boldsymbol{0}$	and 1 from	the least to	the most	significant.	Define a	${\it predicate}$	succ/2	that con	mputes	the
successor o	of a number	in this repre	sentation.	We have for	r example	:				





Problem 5 (1 p.). What is the answer of the Prolog machine to the query

 $?- \ \)$



Problem 6 (2 p.). Write the set of productions of a context-free grammar over the terminal alphabet $\{a,b\}$ and nonterminal alphabet $\{S\}$ that generates the language

$$\{w \in \{a,b\}^* : |w|_a \le |w|_b\},\$$

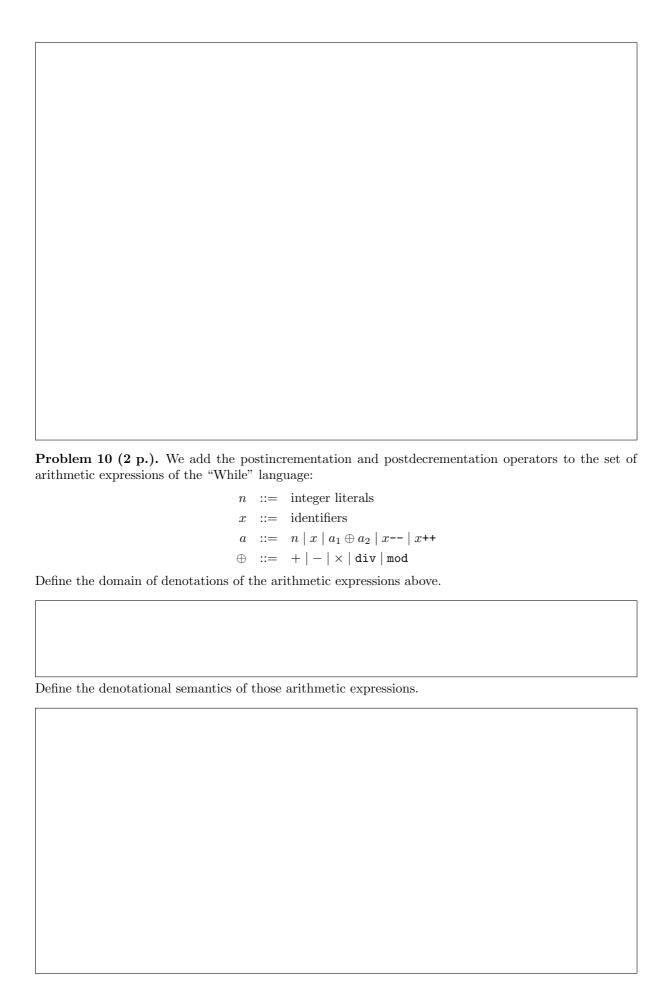
where $|w|_x$ for $x \in \{a, b\}$ stands for the number of occurrences of a symbol x in a word w.

Problem 7 (1 p.). Write a single secontext-free grammar.	entence containing the definition of the notion of an unambiguous
Problem 8 (1 p.). Write the rules instruction	of the big-step operational semantics (natural semantics) for the
	$\mathtt{repeat}\; c\;\mathtt{until}\; b$
Execution of this statement consists when the condition b is satisfied after	in repetitive execution of the statement c . The loop is completed the execution of the statement c .
Problem 9 (1 p.). Arithmetic exprestract syntax:	essions of the "While" language are described by the following ab-
	n ::= integer literals
	x ::= identifiers

Give the rules of the small-step operational semantics (structural operational semantics) for arithmetic expressions of the "While" language.

 $\oplus \ ::= \ + |-| \times | \operatorname{div} | \operatorname{mod}$

 $a ::= n \mid x \mid a_1 \oplus a_2$



Problem 11 (1 p.). Give the weakest precondition for the program
while N \Leftrightarrow O do R := 2 * R; N := N - 1; done
with the postcondition $R = 2^i$.
Problem 12 (3 p.). Decorate the program below with assertions to get the proof of partial correctness of this program with respect to the given specification.
$\{\mathtt{N}=i\}$
<u></u>
{
R := 1;
{
٠
{
while N <> 0 do
{
{
R := 2 * R;

 $\{\mathtt{R}=2^i\}$

done

N := N - 1;

.....

Problem 13 (1 p.). Consider a signature consisting of symbols $a/0$, $b/0$, $z/2$ and $z/2$. Let
$E_1 = \{a + x = a : x, b + x = b : x, (a : x) + y = a : (x + y), (b : x) + y = b : (x + y)\}$
We assume the initial algebra semantics. Prove that $(x + y) + z = x + (y + z)$.